### Calculus

<table>
<thead>
<tr>
<th>Credit Hours</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level</td>
<td>Lower</td>
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</table>

**Exam Code**: 150  
**Catalog Number**: MATx150

The most current content guides are available at:  
www.excelsior.edu/contentguides
Before You Choose This UExcel Exam

Uses for the Examination

- Excelsior College, the test developer, recommends granting four (4) semester hours of lower-level undergraduate credit to students who receive a letter grade of C or higher on this examination.

- Other colleges and universities also recognize this exam as a basis for granting credit or advanced standing.

- Individual institutions set their own policies for the amount of credit awarded and the minimum acceptable score.

Exam-takers who have applied to Excelsior College should ask their academic advisor where this exam fits within their degree program.

Exam-takers not enrolled in an Excelsior College degree program should check with the institution from which they wish to receive credit to determine whether credit will be granted and/or to find out the minimum grade required for credit. Those who intend to enroll at Excelsior College should ask an admissions counselor where this exam fits within their intended degree program.

Examination Length and Scoring

The examination consists of approximately 60 questions, most of which are multiple choice; for samples of all the item types on this exam, see the sample items in the back of this guide. Some items are unscored, pretest items. The pretest items are embedded throughout the exam and are indistinguishable from the scored items. You will have two (2) hours to complete the examination. Your score will be reported as a letter grade.

UExcel Exam Resources

Excelsior College Bookstore

The Excelsior College Bookstore offers recommended textbooks and other resources to help you prepare for UExcel exams.

The bookstore is available online at (login required): www.excelsior.edu/bookstore

UExcel Practice Exams

The official UExcel practice exams are highly recommended as part of your study plan. Once you register for your UExcel exam, you are eligible to purchase the corresponding practice exam, which can be taken using any computer with a supported Web browser. Each practice exam includes two forms that you may take within a 180-day period.

Excelsior College Library

Enrolled Excelsior College students can access millions of authoritative resources online through the Excelsior College Library. Created through our partnership with the Sheridan Libraries of The Johns Hopkins University, the library provides access to journal articles, books, websites, databases, reference services, and many other resources. Special library
pages relate to the nursing degree exams and other selected exams. To access it, visit www.excelsior.edu/library (login is required).

Our library provides:

- 24/7 availability
- The world’s most current authoritative resources
- Help and support from staff librarians

**Online Tutoring**

Excelsior College offers online tutoring through SMARTTHINKING™ to connect with tutors who have been trained in a variety of academic subjects. To access SMARTTHINKING, go to www.excelsior.edu/smarthinking. Once there, you may download a copy of the SMARTTHINKING Student Handbook as a PDF.

**Preparing for UEExcel Exams**

**Take Charge of Your Own Learning**

At Excelsior College, independent, self-directed study supported by resources we help you find is not a new concept. We have always stressed to exam takers that they are acting as their own teacher, and that they should spend as much time studying for an exam as they would spend in a classroom and on homework for a corresponding college course in the same subject area.

Begin by studying the content outline contained in this content guide, at its most detailed level. You will see exactly which topics are covered, and where chapters on those topics can be found in the Recommended Resources. You will see exactly where you might need to augment your knowledge or change your approach.

The content outline, along with the Learning Outcomes for this exam and recommended textbooks, will serve as your primary resources.

**How Long Will It Take Me to Study?**

A UEExcel exam enables you to show that you’ve learned material comparable to one or more 15-week college-level courses. As an independent learner, you should study and review as much as you would for a college course. For a 3-credit course in a subject they don’t know, most students would be expected to study nine hours per week for 15 weeks, for a total of 135 hours.

**Study Tips**

Become an active user of the resource materials. Aim for understanding rather than memorization. The more active you are when you study, the more likely you will be to retain, understand, and apply the information.

The following techniques are generally considered to be active learning:

- **preview or survey** each chapter
- **highlight or underline text** you believe is important
- **write questions or comments** in the margins
- **practice re-stating content** in your own words
- **relate what you are reading** to the chapter title, section headings, and other organizing elements of the textbook
- **find ways to engage** your eyes, your ears, and your muscles, as well as your brain, in your studies
- **study with a partner or a small group** (if you are an enrolled student, search for partners on MyExcelsior Community)
- **prepare your review notes** as flashcards or create recordings that you can use while commuting or exercising

When you feel confident that you understand a content area, review what you have learned. Take a second look at the material to evaluate your understanding. If you have a study partner, the two of you can review by explaining the content to each other or writing test questions for each other to answer. Review questions from textbook chapters may be helpful for partner or individual study, as well.

**Using UEExcel Practice Exams**

We recommend taking the first form of the practice exam when you begin studying, to see how much you already know. After taking the first practice exam, check your performance on each question and find out why your answer was right or wrong. This feedback will help you improve your knowledge of the subject and identify areas of weakness that you should address before taking the exam. Take the second form of the
practice exam after you have finished studying. Analyze your results to identify the areas that you still need to review.

Although there is no guarantee, our research suggests that students who do well on the practice exams are more likely to pass the actual exam than those who do not do well (or do not take advantage of this opportunity).

**About Test Preparation Services**

Preparation for UExcel® exams and Excelsior College® Examinations, though based on independent study, is supported by Excelsior College with a comprehensive set of exam learning resources and services designed to help you succeed. These learning resources are prepared by Excelsior College so you can be assured that they are current and cover the content you are expected to master for the exams. These resources, and your desire to learn, are usually all that you will need to succeed.

There are test-preparation companies that will offer to help you study for our examinations. Some may imply a relationship with Excelsior College and/or make claims that their products and services are all that you need to prepare for our examinations.

Excelsior College is not affiliated with any test preparation firm and does not endorse the products or services of these companies. No test preparation vendor is authorized to provide admissions counseling or academic advising services, or to collect any payments, on behalf of Excelsior College. Excelsior College does not send authorized representatives to a student’s home nor does it review the materials provided by test preparation companies for content or compatibility with Excelsior College examinations.

To help you become a well-informed consumer, we suggest that before you make any purchase decision regarding study materials provided by organizations other than Excelsior College, you consider the points outlined on our website at [www.excelsior.edu/testprep](http://www.excelsior.edu/testprep).

**Preparing for This Exam**

**Prior Knowledge**

A familiarity with precalculus topics including algebra, trigonometry, and functions is assumed.

**Using the Content Outline**

Each content area in the outline includes (1) the recommended minimum hours of study to devote to that content area and (2) the most important sections of the recommended resources for that area. These annotations are not intended to be comprehensive. You may need to refer to other chapters in the recommended textbooks. *Chapter numbers and titles may differ in other editions.*

This content outline contains examples of the types of information you should study. Although these examples are numerous, do not assume that everything on the exam will come from these examples. Conversely, do not expect that every detail you study will appear on the exam. Any exam is only a broad sample of all the questions that could be asked about the subject matter.

**Using the Sample Questions and Rationales**

Each content guide provides sample questions to illustrate those typically found on the exam. These questions are intended to give you an idea of the level of knowledge expected and the way questions are typically phrased. The sample questions do not sample the entire content of the exam and are not intended to serve as an entire practice test.
Recommended Resources for the UExcel Exam in Calculus

The resources and materials listed below were used by the examination development committee to verify all the questions on the exam. Excelsior College recommends you use these resources as the most appropriate information when ordering textbooks from the college's bookstore (see page 1 of this content guide). You should allow ample time to obtain resources and to study sufficiently before taking the exam, so plan appropriately and systematically.

A word about textbook editions: Textbook editions listed in the UExcel content guides may not be the same as those listed in the bookstore. Textbook editions may not exactly match up in terms of table of contents and organization, depending upon the edition. However, our team of exam developers checks exam content against every new textbook edition to verify that all subject areas tested in the exam are still adequately available in the study materials. If needed, exam developers will list supplemental resources to ensure that all topics in the exam are still sufficiently covered. Public libraries may have the textbooks you need, or may be able to obtain them for you through interlibrary loan to reduce textbook costs. You may also consider financial aid, if you qualify, to further help defray the steep cost of textbooks. A section on OER has been included in this guide to help you locate additional resources to augment your study.

Textbook

This textbook was used by the examination development committee to verify all questions on the exam.


These study materials may be purchased from the Excelsior College Bookstore.

Reducing Textbook Costs

Many students know it is less expensive to buy a used textbook, and buying a previous edition is also an option. The Excelsior College bookstore includes a buyback feature and a used book marketplace, as well as the ability to rent digital versions of textbooks for as long as students need them. Students are encouraged to explore these and the many other opportunities available online to help defray textbook costs.

Practice Exam

The Practice Exam is available after you register for this UExcel exam.

A Word About Open Educational Resources

Open educational resources (OER) are educational materials available for study at no cost on the Web. Some OER are available for anyone to access any time. Others, such as Massive Open Online Courses (MOOCs), require sign-up and are only available during certain windows. Please note that some MOOC providers offer certificates of completion or other products or services for a fee. No MOOC or other OER is a complete substitute for the content guide and officially Recommended Resources listed here in this content guide. However, by definition, MOOCs are essentially free of charge and include access to a main body of learning materials that may help you in your learning.

Being an independent learner preparing for credit by exam, you may not need any of the fee-based options that are offered elsewhere online. But if you are looking for a coherent academic course for self-study, lectures on specific topics, or audio or visual materials that fit your learning style better than print materials alone, a MOOC or other type of OER may be your answer. Keep in mind that none of these OER were designed by Excelsior, nor are they guaranteed to match the exam content outlines completely. They are simply another tool available in your study kit.

We highly encourage using the Recommended Resources. In the content outline, you will see that the topics in the exam are referenced to specific portions of recommended textbooks. Using OER alone will not ensure you’ve completely covered the content in the exam, or it may not cover some topics in sufficient-enough depth without the use of the formal, recommended textbooks.

If the OER course you choose does not include a textbook for reference and you do not have significant practical theory-based experience in the field of study,
use a college textbook to ensure adequate preparation for the exam, and use the exam’s content outline as a guide.

Combined with comparable college textbooks, OER provides you with a variety of choices in knowledge sources and learning experiences, to enhance your understanding of the subject matter.

Choosing Open Educational Resources

Most sites for university-based OER can be searched through www.ocwconsortium.org and/or www.oercommons.org.

Sites that specialize in Web courses designed by college professors under contract with the website sponsor, rather than in Web versions of existing college courses, include:

www.education-portal.com
www.opencourselibrary.org (abbreviated as OCL)

We have included specific courses that cover material for one or more UExcel® exams from the sites in the listings above. It’s worth checking these sites frequently to see if new courses have been added that may be more appropriate or may cover an exam topic not currently listed.

In addition, sites like Khan Academy (www.khanacademy.com) and iTunes U feature relatively brief lessons on very specific topics rather than full courses. Full courses are also available on iTunes U (http://www.apple.com/education/ipad/itunes-u/). We have chosen a few courses and collections for this listing.

Other Online Resources

This section of the OER Guide is provided to allow learners to independently search for resources. Send an e-mail to OER@excelsior.edu if you have questions about a resource’s credibility.

Open Online Textbooks
Boundless open textbooks
https://www.boundless.com/open-textbooks/

BookBoon
http://bookboon.com/en/textbooks-ebooks

Flatworld Knowledge
http://catalog.flatworldknowledge.com/#our-catalog
General Description of the Examination

The UExcel Calculus examination is based on material typically taught in a first-semester, four-credit course in calculus.

The examination measures knowledge of terminology, understanding of concepts and theories relating to limits, derivative, and integrals, and the ability to apply this knowledge and understanding in an analysis of problems in business, the sciences, and engineering.

Those beginning to study for this exam should be familiar with concepts covered in precalculus topics including algebra, trigonometry, and functions.

Learning Outcomes

After you have successfully worked your way through the recommended study materials, you should be able to demonstrate the following learning outcomes:

1. Demonstrate both conceptually and computationally an understanding of the three primary topics in Calculus: limits, derivatives, and integrals.
2. Interpret limits based graphically, numerically, and by the application of the basic limit laws and the concept of continuity.
3. Interpret derivatives graphically, numerically, by the definition, and by the application of basic derivative rules.
4. Interpret integrals graphically, numerically, by the definition, and by the application of basic integration rules.
5. Demonstrate understanding of the relationship between definite and indefinite integrals.
6. Apply the three primary topics of Calculus to problems in business, the sciences, and engineering.
**Content Outline**

The content outline describes the various areas of the test, similar to the way a syllabus outlines a course. To fully prepare requires self-direction and discipline. Study involves careful reading, reflection, and systematic review.

The major content areas on the Calculus examination, the percent of the examination, and the hours to devote to each content area are listed below.

<table>
<thead>
<tr>
<th>Content Area</th>
<th>Percent of the Examination</th>
<th>Hours of Study</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Limits and Continuity</td>
<td>15%</td>
<td>27</td>
</tr>
<tr>
<td>II. Derivatives</td>
<td>25%</td>
<td>45</td>
</tr>
<tr>
<td>III. Applications of Derivatives</td>
<td>30%</td>
<td>54</td>
</tr>
<tr>
<td>IV. Integrals</td>
<td>30%</td>
<td>54</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>100%</strong></td>
<td></td>
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</tbody>
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**NOTE:** Occasionally, examples will be listed for a content topic are to help clarify that topic. However, the content of the examination is not limited to the specific examples given.

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**II. Derivatives**

**25 PERCENT OF EXAM | 45 HOURS OF STUDY**

**Chapter 1, Functions**

**Chapter 3, Derivatives**

**A.** Derivative as a slope

**B.** Definition of derivative

**C.** Derivative rules

1. Basic rules
2. Product rule
3. Quotient rule
4. Chain rule

**D.** Derivatives of specific functions

1. Polynomial functions
2. Trigonometric functions
3. Exponential functions
4. Inverse functions (logarithmic and inverse trigonometric)

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**I. Limits and Continuity**

**15 PERCENT OF EXAM | 27 HOURS OF STUDY**

**Chapter 2, Limits and Continuity**

**A.** Finding limits graphically and numerically

1. One-sided limits
2. Two-sided limits
3. Infinite limits
4. Limits at infinity

**B.** Limit laws

**C.** Continuity
III. Applications of Derivatives

30 PERCENT OF EXAM | 54 HOURS OF STUDY

Chapter 1, Functions
Chapter 3, Derivatives
Chapter 4, Applications of Derivatives

A. Implicit differentiation
   1. Related rates
   2. Equation of the line tangent to the curve

B. Graphing
   1. Critical values
   2. Increasing/decreasing
   3. Maxima and minima (global and local)
   4. Concavity
   5. Inflection points
   6. Asymptotes (vertical and horizontal)

C. Rolle's Theorem and Mean Value Theorem

D. Position, velocity, and acceleration

E. Applied optimization problems

F. L'Hôpital's Rule

G. Differential and linear approximation

IV. Integrals

30 PERCENT OF EXAM | 54 HOURS OF STUDY

Chapter 5, Integrals
Chapter 6, Applications of Definite Integrals

A. Finite sums and sigma notation
B. Riemann sums and approximate integration
C. Definite integrals as signed area
D. Indefinite integrals
E. Fundamental theorem of calculus
F. Integration by substitution
G. Areas between curves
H. Average value of the function
I. Applications (for example: areas, volumes, lengths, work, pressure, force)
The sample questions give you an idea of the level of knowledge expected in the exam and how questions are typically phrased. They are not representative of the entire content of the exam and are not intended to serve as a practice test. Rationales for the questions can be found on pages 13–17 of this guide. In that section, the correct answer is identified and each answer is explained. The number in parentheses at the beginning of each rationale refers to the corresponding section of the content outline. For any questions you answer incorrectly, return to that section of the content outline for further study.

You will be provided with an erasable white board to use during your exam. Necessary formulas will be loaded on your PC as an exhibit for your reference, and you will have access to a basic 8-function calculator on the computer. The calculator button is in the top left hand corner of the page as each question is presented and the tables and formulas are in the Help button located in the lower left hand corner as each question is presented. A copy of the formulas and a picture of a typical 8-function calculator are provided at the back of this content guide.

1. What is the value of \( \lim_{x \to 1} \frac{x^3 - 1}{x - 1} \)?
   1) 0
   2) 1
   3) 3
   4) \( \infty \)

2. Base your answer to the following question on the graph of \( y = f(x) \) shown below:

   ![Graph of f(x)](image)

   What is the value of \( \lim_{x \to 2} f(x) \)?

3. Base your answer to the following question on the graphs of \( f(x) \) and \( g(x) \) below:

   ![Graphs of f(x) and g(x)](image)

   What are the values of the following limits?
   
   I = \( \lim_{x \to a} f(x) \), II = \( \lim_{x \to a} f(x) \), III = \( \lim_{x \to a} f(x) \), IV = \( \lim_{x \to a} f(x) \)
   
   1) (I, II, III, IV) = (+\( \infty \), -\( \infty \), 1, 2)
   2) (I, II, III, IV) = (-\( \infty \), +\( \infty \), 1, 2)
   3) (I, II, III, IV) = (+\( \infty \), -\( \infty \), 2, 1)
   4) (I, II, III, IV) = (-\( \infty \), +\( \infty \), 2, 1)
4. Base your answer to the following question on the graph of \( f(x) \) below:

Which is the best estimate for \( f'(2) \) using the graph?

1) –2
2) –0.6
3) 0.6
4) 2

5. What is the derivative for the function \( f(x) = [\sin^2(x)][\sin(x^3)] \)?

1) \( 2\sin(x)[x\cos(x^2)\sin(x) + \cos(x)\sin(x^2)] \)
2) \( \cos^2(x)\cos(x^3) \)
3) \( 2\cos(x)[\sin(x^2)\cos(x) + \sin(x)\cos(x^2)] \)
4) \( 2\sin(x)\sin(x^2)[\sin(x) + \cos(x)] \)

6. Which is derivative for the function \( f(x) = \frac{e^x}{2x} \)?

1) \( \frac{e^x}{2x} + \frac{e^{2x}}{2x^2} \)
2) \( e^x - \frac{e^{2x}}{2x^3} \)
3) \( e^x - \frac{e^{2x}}{2} \)
4) \( e^x + \frac{e^{2x}}{2} \)

7. What is the derivative of \( f(x) = \sin(1 + ax^3) \), where \( a \) is constant?

1) \( \cos(1 + ax^3) \)
2) \( \cos(3ax^3) \)
3) \( 3ax^2\cos(1 + ax^3) \)
4) \( -3ax^2\cos(1 + ax^3) \)

8. Given \( f(x) = kx^3 + 4x^2 - 3x + 2 \), what is the value of \( k \) such that \( f''(2) = 25 \)?

1) 1
2) \( -\frac{5}{8} \)
3) \( \frac{5}{8} \)

9. The area of a circle is increasing at a constant rate of \( 64\pi \, \text{cm}^2/\text{s} \). What is the rate of change for the radius when the circle has an area of \( 36\pi \, \text{cm}^2 \)?

10. Which is the equation of the tangent line to \( y = 1 + \ln(x) \) at \( x = 1 \)?

1) \( y = 2 - \frac{1}{x} \)
2) \( y = x \)
3) \( y = \frac{1}{x} \)
4) \( y = x - 2 \)

11. Base your answer to the following question on the graph below showing three functions (I, II, and III):

Which functions correspond to \( f, f', \) and \( f'' ?

1) \( f = \text{I}, f' = \text{II}, f'' = \text{III} \)
2) \( f = \text{I}, f' = \text{III}, f'' = \text{II} \)
3) \( f = \text{II}, f' = \text{III}, f'' = \text{I} \)
4) \( f = \text{II}, f' = \text{I}, f'' = \text{III} \)
12. Base your answer to the following question on the graphs below:

A function $f(x)$ has the following conditions:

- $f'(x) > 0$ for $-\infty < x < -2$
  $-1 < x < 1$
  $1 < x < \infty$
- $f'(x) < 0$ for $-2 < x < -1$
- $f''(x) > 0$ for $-\infty < x < -3$
  $0 < x < 1$
- $f''(x) < 0$ for $-3 < x < -1$
  $-1 < x < 0$
  $1 < x < \infty$

Which graph represents the function $f(x)$?

1) I
2) II
3) III
4) IV

13. Base your answer to the following question on the graph of the function $f(x)$ below:

Which is the best estimate of the value $c$ so that $f'(c) = \frac{f(5) - f(1)}{5 - 1}$?

1) 1
2) 2
3) 3
4) 4
14. An open (no lid) rectangular box is constructed so that the length of the base is three times longer than the width of the base. What is the minimum surface area of a box that holds 18 ft³ in volume?

\[ \text{ft}^2 \]

15. What is the value of \( \int_{0}^{4} 3x \, dx \)?

1) -2.5
2) -6.5
3) 6.5
4) 13

16. Base your answer to the following question on the graph of function \( f(x) \) below:

\[ f(x) \]

\[ y \]

\[ x \]

\[ 4 \]

\[ 2 \]

\[ -2 \]

\[ -4 \]

\[ f(x) \]

I. \( \int_{0}^{4} f(x) \, dx = -8 \)
II. \( \int_{-4}^{0} f(x) \, dx = -8 \)
III. \( \int_{-4}^{4} f(x) \, dx = 0 \)

Which statement(s) is true?

1) II only
2) III only
3) I and II only
4) I and III only

17. Let \( f(x) = \int_{0}^{x^3} \sin(t^3) \, dt \).

What is the result of \( \frac{d}{dx} f(x) \)?

1) \( \sin(x^8)2x \)
2) \( \sin(x^5) \)
3) \( \sin(x^8)2x \)
4) \( \sin(x^6) \)

18. What is the value of the definite integral \( \int_{1}^{\ln(x)} \frac{dx}{x} \)?

1) 1
2) \( \frac{1}{e} - 1 \)
3) \( \frac{1}{2}(e^2 - 1) \)
4) \( \frac{1}{2} \)

19. A particle moves with a velocity of \( v(t) = \sin(t) \) for \( 0 \leq t < \frac{\pi}{2} \). If the particle is at a position of 2 at \( t = 0 \), what is the position of the particle at \( t = \frac{\pi}{4} \)?

1) \( \frac{\sqrt{2}}{2} + 2 \)
2) \( \frac{\sqrt{2}}{2} + 1 \)
3) \( 3 - \frac{\sqrt{2}}{2} \)
4) \( 1 - \frac{\sqrt{2}}{2} \)

20. The function \( f(x) = \frac{1}{x^2} \) has an average of \( \frac{1}{4} \) on the interval \([1, c], c > 1\). What is the value of c?

1) \( \frac{1}{2} \)
2) \( \frac{4}{3} \)
3) 2
4) 4
Rationales

1.(I.A.2)
1) You substituted 1’s for x’s in the equation and ended up with 0/0. You incorrectly interpreted the indeterminate form as zero.
2) You ignored the “–1” term in both the numerator and denominator and ended up with 1/1 instead.
*3) SOLUTION:
Factor the polynomial:
\[ x^3 - 1 = (x - 1)(x^2 + x + 1) \]
\[ \lim_{x \to 1} \frac{x^3 - 1}{x - 1} = \lim_{x \to 1} x^2 + x + 1 = 3 \]
4) You incorrectly evaluated the limit as x approaches infinity.

2.(I.A.2)
*1) SOLUTION:
As the limit approaches x = 1 from the left, \( f(x) \) approaches infinity (+∞).
As the limit approaches x = 1 from the right, \( f(x) \) approaches negative infinity (−∞).
As the limit approaches x = 3 from the left, \( f(x) \) approaches 1.
As the limit approaches x = 3 from the right, \( f(x) \) approaches 2.
2) If you work backwards, left/right, you get this answer.
3) If you reverse +/- for x \( \to 3 \) but not for x \( \to 1 \), you get this answer.
4) If you reverse +/- for x \( \to 1 \) but not x \( \to 3 \), you get this answer.

3.(I.I.B)
*1) SOLUTION:
From the graph of \( f(x) \), the limit at x approaches 2 is 2. For \( g(x) \), the limit at x approaches 2 is also 2. Therefore,
\[ \lim_{x \to 2} \frac{f(x)}{g(x)} = \lim_{x \to 2} \frac{f(x)}{g(x)} = \frac{2}{2} = 1 \]

4.(II.B)
1) The slope is positive at x = 2.
2) The slope is positive at x = 2.
*3) SOLUTION:
\[ f'(2) \text{ is the slope of the function at } f(2). \]
Slope \( = \frac{7}{5} \times \frac{4}{9} = \frac{28}{45} \approx 0.6 \)
4) You mistakenly evaluated the function at x = 2 instead of the slope at x = 2. \( f'(2) = 2 \)
5.(II.C.2)  
*1) SOLUTION:  
Using the Product Rule:  
\[ f'(x) = \sin^2(x) \frac{d}{dx} [\sin(x^2)] + \sin(x^2) \frac{d}{dx} [\sin(x^2)]  
\]
\[ f'(x) = \sin^2(x) [\cos(x^2)](2x) + \sin(x^2)[2 \sin(x) \cos(x)]  
\]
\[ f'(x) = 2 \sin(x) [x \cos(x^2) + \cos(x) \sin(x^2)]  
\]
2) You did not apply either the Product Rule or the Chain Rule.  
3) You used the Product Rule but did not apply the Chain Rule.  
4) You forgot to differentiate \( \sin(x^2) \) in the Product Rule.  

6.(II.C.3)  
1) You made a sign error while calculating \( f'(x) \).  
*2) SOLUTION:  
The answer is fairly non-intuitive unless you follow the quotient rule formula:  
\[ \left( \frac{f(x)}{g(x)} \right)' = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2} = \frac{(2x)^2 e^{x^2} - e^{x^2}}{(2x)^2}  
\]
3) You made a sign error calculating \( f'(x) \) and used \( g'(x) \) instead of \( g(x) \).  
4) You used \( g'(x) \) instead of \( g(x) \).  

7.(II.C.4)  
1) You made the common error of forgetting to take the derivative inside the cosine function.  
2) You took the derivative inside the cosine function only.  
*3) SOLUTION:  
\[ f'(x) = \frac{d}{dx} [\sin(1 + ax^3)] = \frac{d}{dx} [1 + ax^3] \cos(1 + ax^3) = 3ax^2 \cos(1 + ax^3)  
\]
4) You confused the derivative of sine as negative cosine.  

8.(II.D.1)  
*1) SOLUTION:  
\[ f'(x) = 3kx^2 + 8x - 3  
\]
\[ f'(2) = 3k(2)^2 + 8(2) - 3 = 25  
\]
\[ 12k = 25 - 16 + 3 = 12  
\]
\[ k = 1  
\]
2) You made a sign error.  
3) You evaluated \( f(2) \) instead.  
4) You evaluated \( f(2) \) instead and made a sign error.  

9.(III.A.1)  
*SOLUTION:  
\[ a = \pi \ r^2, \quad \frac{dA}{dt} = 64\pi, \quad \frac{dr}{dt} = ?  
\]
When \( A = 36\pi, \ 36\pi = \pi \ r^2, \) and \( r = 6 \)  
\[ \frac{dA}{dt} = 2\pi \ r \frac{dr}{dt}  
\]
\[ 64\pi = 2\pi \ (6) \frac{dr}{dt}  
\]
\[ \frac{16}{3} = \frac{dr}{dt}  
\]

10.(III.A.2)  
1) You made a computational error as follows:  
\[ y - 1 = \frac{1}{x} (x - 1) = 1 - \frac{1}{x}, \quad y = 2 - \frac{1}{x}  
\]
*2) SOLUTION:  
\[ y = 1 + \ln 1 = 1 + 0 = 1  
\]
\[ \frac{dy}{dx} = \frac{1}{x}  
\]
\[ m = \frac{1}{1} = 1  
\]
\[ y - 1 = 1(x - 1)  
\]
\[ y = x  
\]
3) You only calculated the derivative of the function.  
4) You made a simple computational error.  

*correct answer
11.(III.B) 1) Solution:
Function I is above the x-axis, so it cannot be the derivative of any of the other two functions. If it was, then the function would be always increasing. Wherever function I has an extremum, function II crosses the x-axis. This means that function II is a derivative of function I. Similar analysis shows that function III is a derivative of function II.

\[ I = (x-1)^2(x+1)^2, \quad II = \frac{d}{dx} I, \quad III = \frac{d}{dx} II \]

2) Function III cannot be a derivative function of function I and function II cannot be a derivative function of function III.

3) Function III is a derivative function of function II but function I cannot be a derivative function of function III.

4) Function I cannot be a derivative function of function II and function III cannot be a derivative function of function I.

12.(III.B) 1) SOLUTION:
- Functions I and II satisfy the conditions for \( f'(x) < 0 \).
- All four functions satisfy the conditions for \( f'(x) < 0 \).
- Functions I and IV satisfy the conditions for \( f''(x) < 0 \).
- Functions I and IV satisfy the conditions for \( f'''(x) < 0 \).

2) Function III does not satisfy the 1st, 3rd and 4th conditions.

3) Function II does not satisfy the 3rd and 4th conditions.

4) Function IV does not satisfy the 1st condition.

13.(III.C) 1) You incorrectly estimated the slope.

2) SOLUTION:
The slope at \( x = 3 \) is the same as the slope of the line segment connecting (1.5, 0) and (4.5, 6). The slope is approximately equal to 2.

3) You mistakenly evaluated the function at \( x = 2 \) instead of finding the slope.

4) You incorrectly estimated the slope.

14.(III.E) SOLUTION:
\[ V = 3x^2y = 18, \quad y = \frac{6}{x^2} \]

\[ A = 3x^2 + 2xy + 6xy = 3x^2 + 8xy = 3x^2 + 8x\left(\frac{6}{x}\right) = 3x^2 + \frac{48}{x} \]

\[ \frac{dA}{dx} = 6x - \frac{48}{x^2} = 0, \quad \frac{6x}{1} = \frac{48}{x^2}, \quad 6x^3 = 48, \quad x^3 = 8, \quad x = 2 \]

\[ \frac{d^2A}{dx^2} = 6 + \frac{96}{x^3}, \quad \text{at} \; x = 2 \text{ ft}, \quad \frac{d^2A}{dx^2} = 18 > 0, \quad \therefore x = 2 \text{ is a minimum}. \]

Surface Area = \[ 3(2)^2 + \frac{48}{2} = 12 + 24 = 36 \text{ ft}^2 \]
15. (IV.C)  
1) \[ \int_{0}^{5} x^2 - 3\,dx = \frac{1}{2} x^2 - 3x^2 \bigg|_{0}^{5} = \frac{25}{2} - 15 = \frac{5}{2} \]  
2) You made a sign error: \[ \frac{9}{2} + 2 = -\frac{1}{2} \]  
*3) SOLUTION:

*correct answer

Graph of \( y = x + 3 \)

Integral Area = \( \frac{1}{2} (3)(3) + \frac{1}{2} (2)(2) = \frac{9}{2} + 2 = \frac{13}{2} \)

4) You forgot to divide by two for area of triangle: \( (3)(3) + (2)(2) = 13 \)

16. (IV.C)  
1) You confused \(+/-\) areas.  
2) You failed to see that statement I is also true.  
3) You didn’t understand that switching the limits of integration changes the sign of the integral.  
*4) SOLUTION:

Statement I is true:

\[ \int_{\frac{3}{4}}^{4} f(x)\,dx = \int_{0}^{\frac{3}{4}} f(x)\,dx + \int_{\frac{3}{4}}^{2} f(x)\,dx + \int_{2}^{4} f(x)\,dx = -8 + (-2) + 2 = -8 \]

Statement II is false:

\[ \int_{0}^{\frac{3}{4}} f(x)\,dx = \int_{0}^{2} f(x)\,dx = -(-8) = 8 \]

Statement III is true:

\[ \int_{0}^{\frac{3}{4}} f(x)\,dx = \int_{0}^{2} f(x)\,dx + \int_{2}^{4} f(x)\,dx = -2 + 2 = 0 \]

17. (IV.E)  
*1) SOLUTION:

\[ g(x) = \int_{0}^{x} \sin(t^3)\,dt, \quad g'(x) = \sin(x^3) \]

\[ h(x) = x^2, \quad h'(x) = 2x \]

\[ f(x) = g(h(x)) \]

\[ f'(x) = g'(h(x))h'(x) = \sin((x^2)^3)2x \]

\[ f'(x) = \sin(x^3)2x \]

2) You used a bad application of the Fundamental Theorem of Calculus  
3) You applied the Fundamental Theorem of Calculus without using the Chain Rule.  
4) You failed to use the Chain Rule.

18. (IV.F)  
1) You forgot to carry 1/2 in the final calculation.  
2) You forgot to take the natural logs when evaluating and forgot to carry 1/2 in the calculation.  
3) You forgot to take the natural logs when evaluating the definite integral.  
*4) SOLUTION:

\[ u = \ln x, \quad du = \frac{dx}{x} \]

\[ \int_{\frac{1}{2}}^{1} \frac{\ln(x)}{x}\,dx = \int u\,du = \frac{u^2}{2} + C = \frac{[\ln(x)]^2}{2} + C \]

\[ \int_{\frac{1}{2}}^{1} \frac{[\ln(x)]^2}{2}\,dx = \frac{1}{2} - 0 = \frac{1}{2} \]

19. (IVF)  
1) You forgot to integrate the function.  
2) You incorrectly integrated the sine function.  
*3) SOLUTION:

\[ p(t) = \int_{0}^{\frac{\pi}{2}} \sin(t)\,dt + p(0) = 2 \]

\[ p\left(\frac{\pi}{2}\right) = \int_{0}^{\frac{\pi}{2}} \sin(t)\,dt + p(0) = \frac{\pi}{2} \sin(t)\,dt + 2 \]

\[ p\left(\frac{\pi}{2}\right) = -\cos(t)\bigg|_{0}^{\frac{\pi}{2}} + \frac{\pi}{2} \sin(t)\,dt + 2 = 3 - \frac{\sqrt{2}}{2} \]

4) You omitted the constant of integration.
20.(IVH)

1) You integrated the function incorrectly.

2) You forgot to divide by \( c - 1 \).

3) You divided by \( c \) instead of \( c - 1 \).

*4) SOLUTION:

\[
\text{avg val} = \frac{1}{c-1} \int_{1}^{c} \frac{1}{x^2} \, dx = \frac{1}{c-1} \int_{1}^{c} x^{-2} \, dx = \frac{1}{c-1} \left[ x^{-1} \right]_{1}^{c} = \frac{1}{c-1} \left( \frac{1}{c} - 1 \right) = \frac{1}{c-1} \left( 1 - \frac{1}{c} \right)
\]

\[
= \frac{1}{c-1} \left( \frac{c-1}{c} \right) = \frac{1}{c}
\]

thus \( \frac{1}{c} = \frac{1}{4} \), \( c = 4 \)

*correct answer
Formula Sheet

\[
\int \frac{du}{\sqrt{1 - u^2}} = \sin^{-1}(u) + C \\
\int -\frac{du}{\sqrt{1 - u^2}} = \cos^{-1}(u) + C \\
\int \frac{du}{1 + u^2} = \tan^{-1}(u) + C \\
\int -\frac{du}{1 + u^2} = \cot^{-1}(u) + C \\
\int \frac{du}{|u|\sqrt{u^2 - 1}} = \sec^{-1}(u) + C \\
\int -\frac{du}{|u|\sqrt{u^2 - 1}} = \csc^{-1}(u) + C
\]

Volume

<table>
<thead>
<tr>
<th>Shape</th>
<th>Volume Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sphere</td>
<td>( V = \frac{4}{3}\pi r^3 )</td>
</tr>
<tr>
<td>Cylinder</td>
<td>( V = \pi r^2 h )</td>
</tr>
<tr>
<td>Cone</td>
<td>( V = \frac{1}{3}\pi r^2 h )</td>
</tr>
<tr>
<td>Prism with Parallel Bases</td>
<td>( V = Bh )</td>
</tr>
</tbody>
</table>

Area

<table>
<thead>
<tr>
<th>Shape (surface)</th>
<th>Area Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sphere</td>
<td>( A = 4\pi r^2 )</td>
</tr>
<tr>
<td>Cylinder</td>
<td>( A = 2(\pi r^2) + 2\pi rh )</td>
</tr>
<tr>
<td>Cone (surface)</td>
<td>( A = \pi r^2 + 2\pi rl )</td>
</tr>
<tr>
<td>Rectangle</td>
<td>( A = bh )</td>
</tr>
</tbody>
</table>
SECTION FIVE

Taking the Exam

Registering for Your Exam

Register Online

www.excelsior.edu/examregistration
Follow the instructions and pay by Visa, MasterCard, American Express, or Discover Card.

Examination Administration

Pearson Testing Centers serve as the administrator for all Excelsior College computer-delivered exams. The Disability Services office at Excelsior College is responsible for considering requests for reasonable accommodations (exceptions for individual students with documented disabilities). If you are requesting an accommodation due to a disability, download and complete a Request for Accommodation form that can be accessed by visiting the Excelsior College website at www.excelsior.edu/disability-services.

Computer-Delivered Testing

You will take the exam by computer, entering your answers using either the keyboard or the mouse. The system is designed to be as user-friendly as possible, even for those with little or no computer experience. On-screen instructions are similar to those you would see in a paper examination booklet.

Before taking your exam, we strongly encourage you to go on a virtual tour of the testing center. To access this tour, click the What to Expect in a Pearson VUE test center at the following link: home.pearsonvue.com/test-taker/security.aspx

You also will receive a small, erasable whiteboard if you need one. You may not take your own calculator, if the exam calls for it. One will be provided on the testing screen. See example below

On the Day of Your Exam

Important Reminders

On the day of your exam, remember to:

• dress comfortably: the computer will not mind that you’re wearing your favorite relaxation outfit
• arrive at the test site rested and prepared to concentrate for an extended period
• allow sufficient time to travel, park, and locate the test center
• be prepared for possible variations in temperature at the test center due to weather changes or energy conservation measures
bring your ID, but otherwise, don’t weigh yourself down with belongings that will have to be kept in a locker during the test.

**Academic Honesty Nondisclosure Statement**

- All test takers must agree to the terms of the Excelsior College Academic Honesty Policy before taking an examination. The agreement will be presented on screen at the Pearson VUE Testing Center before the start of your exam.
- Once the test taker agrees to the terms of the Academic Honesty Nondisclosure Statement, the exam will begin.

**If you choose not to accept the terms of the agreement**

- your exam will be terminated
- you will be required to leave the testing center
- you will not be eligible for a refund. For more information, review the Student Policy Handbook at [www.excelsior.edu/studentpolicyhandbook](http://www.excelsior.edu/studentpolicyhandbook).

Student behavior is monitored during and after the exam. Electronic measures are used to monitor the security of test items and scan for illegal use of intellectual property. This monitoring includes surveillance of Internet chat rooms, websites, and other public forums.

**Information About UExcel Exams for Colleges and Universities**

A committee of teaching faculty and practicing professionals determines the learning outcomes to be tested on each exam. Excelsior College Center for Educational Measurement staff oversee the technical aspects of test construction in accordance with current professional standards. To promote fairness in testing, we take special care to ensure that the language used in the exams and related materials is consistent, professional, and user friendly. Editorial staff perform systematic quantitative and qualitative reviews to ensure accuracy, clarity, and compliance with conventions of bias-free language usage.

Excelsior College, the test developer, recommends granting four (4) semester hours of lower-level undergraduate credit to students who receive a letter grade of C or higher on this examination. Other colleges and universities also recognize this exam as a basis for granting credit or advanced standing.

Individual institutions set their own policies for the amount of credit awarded and the minimum acceptable grade.

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